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A STUDY ON THE TIDE REACHING SAINT JOHN, NEW BRUNSWICK, IN THE BAY OF FUNDY

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ABSTRACT

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We review in the present study, the results of annual analyses for Saint John covering the years 1894 to 1917 and from 1947 to 1988. The first set of results could be recuperated from Doodson's 1924 paper while the latter set of observations is accessible to computer processing. It was impossible to evaluate the intermediate years because the data is not yet digitized. The tide gauge at Saint John Harbour (45°19'N 65°59'W), in the Bay of Fundy, has functioned almost continuously since 1894. The data scrutinized by Doodson (1924) as well as the one covering 1947–1979, reveals that the amplitude of the major components of the tide, M2, has been increasing since the beginning of the observations. The amplification is of the order of 10 to 15 cm/century and shows signs of accelerating. The solar component S_2, on the other hand, is diminishing at the rate of 4 to 5 cm/century. Since the conventional nodal corrections fail to stabilize the M_2 component, Doodson developed a formula to reproduce its local 19 year modulations. The formula took account of the effects of quadratic friction and revealed the phase discontinuity between M_2 and its satellites, but it overlooked the reduction in the size of the nodal modulations. A least square fit over a set of M_2 samples, derived from the later years, gives an actual range of modulation of $\pm$2.8% with a possible error of at least $\pm$0.7%, compared with the theoretical $\pm$3.7%. Consequently the tide in the Bay of Fundy, known to be strongly affected by frictional effects, is also evolving rapidly, probably because of the continuous increase in sea level and a redistribution of the sediments in Minas and Cumberland Basins.

RÉSUMÉ

Godin, G. 1991. A study on the tide reaching Saint John, New Brunswick, in the Bay of Fundy. Can Contract. Rep. Hydrogr. Ocean Sci. 40: 33 p.

La présente étude porte sur les résultats d'analyses annuelles de données recueillies à Saint John, de 1894 à 1917 et de 1947 à 1988. Le première série de données a été relevée dans le document de Doodson de 1924; la seconde série est informatisée. Les données concernant la période intermédiaire n'ayant pas encore été informatisées, elles n'ont pu faire partie de l'étude. Le marégraphe du port de Saint John (45°19'N 65°59'W), du côté de la baie de Fundy, fonctionne presque sans arrêt depuis 1894. Les données examinées par Doodson et celles qui couvrent la période de 1947 à 1979 révèlent que l'amplitude du principal élément de la marée, M_2, augmente depuis le début des observations de 10 à 15 cm par siècle, et que cette tendance s'accélère apparemment. L'élément solaire S_2, en revanche, diminue à raison de 4 ou 5 cm par siècle. À défaut

d'avour pu stabiliser l'élément M_2 à l'aide des corrections nodales classiques, Doodson a conçu une formule permettant de reproduire ses modulations locales de 19 ans. Cette formule, qui tient compte des effets du frottement quadratique, révèle une rupture de phase entre M_2 et ses satellites; cependant, elle ne tient pas compte de la diminution d'amplitude des modulations nodales. En appliquant la méthode des moindres carrés à un groupe d'échantillons de valeurs M_2 des dernières années, on obtient une fourchette de modulation de $\pm$ 2,8%, avec une marge d'erreur d'au moins $\pm$ 0,7%; la fourchette théorique est de $\pm$ 3,7%. Par conséquent, la marée, dans la baie de Fundy, réputée subir lourdement les effets du frottement, connait une évolution rapide, due probablement à l'augmentation constante du niveau de l'océan et à une redistribution des sédiments dans le bassin Minas et le bassin de Cumberland.

INTRODUCTION

A tide gauge was established in 1894 by W.B. Dawson at Saint John, New Brunswick, according to British standards. Its accumulating records are now becoming an increasingly valuable geophysical series. Most of it however, remains in manuscript form and is still unaccessible to modern data processing; only the data later than 1947 has yet been digitized. The Saint John data, when it first became available, was submitted to a sequence of one year harmonic analyses on a routine basis, by the British Admiralty. It became obvious with time, that the calculated harmonic constants exhibited unexpected fluctuations which defied any simple explanation. W.B. Dawson brought these difficulties to the attention of A.T. Doodson of the Liverpool Tidal Institute, who scrutinized the analyses covering the years 1894 to 1917. Doodson summarized his findings in a paper (Doodson 1924) which also was concerned with similar problems experienced with the harbour of Bombay, India. We show in the inset below a facsimile of the title page and abstract of Doodson's paper; we underline the statement about the secular change in the range of the tide at Saint John and Bombay.

Perturbations of Harmonic Tidal Constants.

By A. T. Doodson, Tidal Institute, University of Liverpool.

(Communicated by H. Lamb, F.R.S.—Received June 14, 1924.)

An examination of the results of analyses of tidal records, as expressed in the so-called " harmonic constants " has yielded some very interesting results, which are discussed in this paper. The author calls attention especially to the influences of tidal friction, but he also points out that one of the perturbations examined has importance in connection with the dynamical theory of tides in oceans. Some interesting secular changes are also discussed, certain evidence showing that the tides are diminishing in range at some places and increasing in range at other places ; changes in the basins of the seas are suggested in explanation.

The paper brought important new insights to tidal research and showed that:

1) Tidal data covering numerous years can reveal the possibility of changes in the local tide

2) Third order modulations are detectable in some of the components which they affect

3) The analytically intractable factor $u|u|$ in the quadratic friction term can actually be approximated very closely by the nondimensional function $au+bu^3$, a and b being suitable constants, thus allowing to deduce the physical consequences of quadratic friction on the harmonic components of the tide.

In the specific case of Saint John, Doodson deduced that the amplitude of the M_2 tide (1 cycle/12.42h) was diminishing with time. Longer series of data however show that the situation is the opposite, that the long range trends point to an increase in the range (twice the amplitude) of M_2.

We show in the following inset a facsimile' of another portion of Doodson's paper concerned with the presence of third order effects in N_2.

---

and the term noted in N_2 is the largest of this latter class. Consequently N_2 contains two terms whose speeds are nearly equal and the forces giving rise to them have different geographical distributions. The amplitudes and phase lags for the principal term (N_2) and the subsidiary term will be denoted respectively by suffixes zero and unity. The results of analysis are as follows :—

	St. John.	Bombay.
H_0	2·030 feet	0·989 feet.
H_1	0·122 „	0·025 „
κ_0	295°	315°
κ_1	233°	208°
H_1/H_0	0·060	0·025
$\kappa_1 - \kappa_0$	−62°	−107°

(The Bombay results are much better than the St. John results; there is a conjugate term present in the St. John results.)

Doodson had calculated in his development of the potential of the tide generating forces (Doodson 1921), small but not completely negligible third order contributions to some of the harmonic components of the Moon. These originate from an additional correction term to the field of gravity, due to the tidal forces exerted by the Moon. Nine year modulations were obvious in the amplitude and phase lag of the component N_2 (1 cycle/12.66h) at Saint John; Doodson could identify these, for the first time, as actual third order effects which till then (Lévy 1898), were considered as being beyond the reach of measurements. He detected a third order contribution to N_2 of 3.7 cm while the observations covering 1947-1979 (Godin 1988) give 3.2cm, a reasonable agreement. (We note that 2.030 ft amounts to 61.9 cm; the Saint John data covering 1947-1979 yields an average N_2 amplitude of 62.0cm. 0.122 ft gives a third order contribution of 3.7 cm compared to 3.2 cm for the later series).

Finally he developed the approximation $u\,|\,u\,| \approx au + bu^3$ to deduce the existence of a conjugate term modulating M_2, having a phase lag quite different from that of M_2. He applied the same approximation in a later contribution on storm surges (Doodson 1956). This type of compact and accurate approximations was called "parexic" by Lanczos (1956) and forms the basis of modern numerical calculus. Dronkers (1964) formalized Doodson's approximation to $u\,|\,u\,|$ in terms of Chebyshev polynomials: it proved itself to be immensely useful for the understanding of frictional effects on the propagation of tides (Godin 1989, 1990, 1991). Therefore Doodson's classical paper on the peculiarities of the tide at Saint John opened the door to modern tidal research.

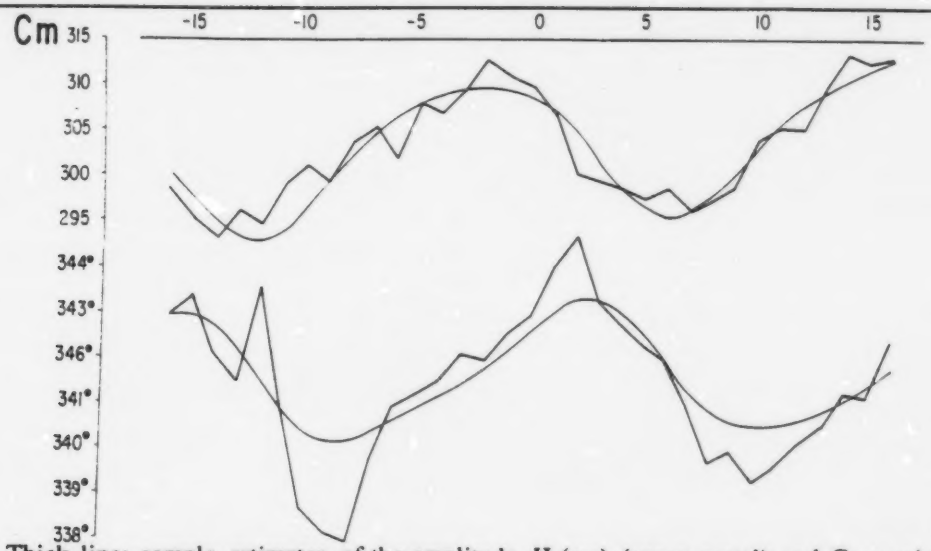


Figure 1. Thick line: sample estimates of the amplitude H (cm) (upper panel) and Greenwich phase lag g (degrees) of M_2, derived from a succession of one year analyses on the water level records at Saint John, covering the interval 1947-1979

Fine line: least square fit to the vector (H,g) by the function

$$(303.3+0.145y, 341.84°+0.072°y)+(7.67, dy-324°)+(1.28, d_1y-245°) \quad \text{cm}$$

where $d=1$ cycle/18.6years and $d_1=1$ cycle/8.9years.

-16 corresponds to the year 1947 on the horizontal time scale

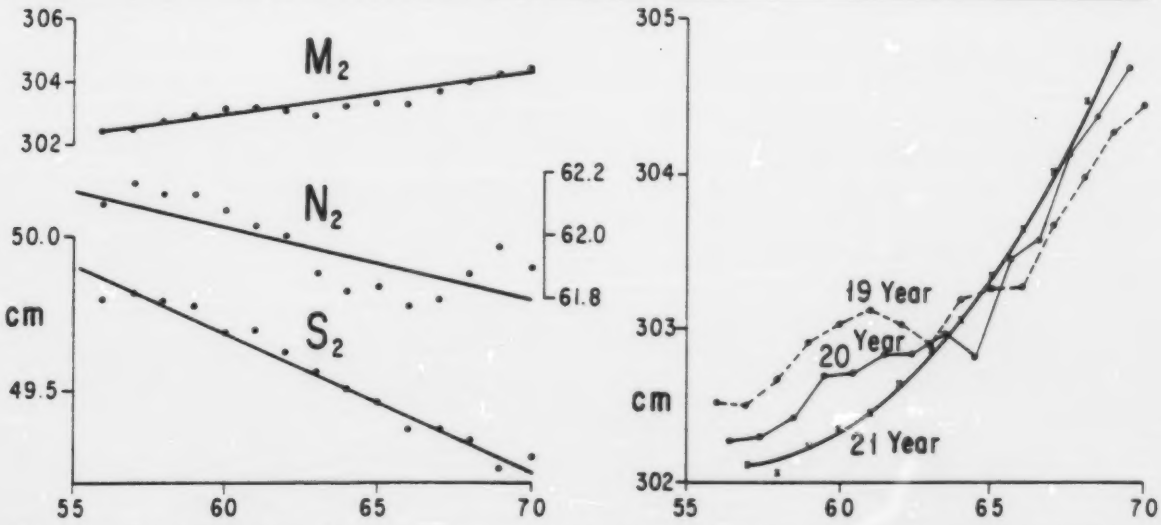


Figure 2. Left hand side: 19 year running means on annual samples on the amplitude (cm) of the major components of the semidiurnal tide at Saint John, New Brunswick, centered over the years 1956 to 1970.

Right hand side: application of 19,20 and 21 year running means to the annual samples of the M_2 amplitude at Saint John, in order to remove the periodicities which obscure the rate of increase

SECULAR TRENDS IN THE AMPLITUDE AND PHASE LAG OF THE M_2 COMPONENT

Doodson's work on the unique peculiarities of the tide at Saint John and the problems it presented was eventually pursued in Mexico (Godin and Gutierrez 1986, Godin 1988). The salient point is that M_2 is the most variable component in the local tide (Fig.1). The application of the conventional nodal corrections f to the amplitude, simply reverses its cycles in amplitude, indicating that they are excessive: this is because the local nodal modulations are reduced by friction, as shown in the studies just mentioned. The variability of the M_2 component, however, is such that it becomes impossible to calculate with any accuracy from the observed time series shown in Fig.1, the empirical values of its modulations.

We wish first to check on the secular trends noted by Doodson in the amplitude of M_2; all we can use, for this purpose, is his report and the data available since 1947. The task of extracting trends in such a highly variable quantity, endowed on top of this by marked 19 year periodicities, is therefore difficult. The only practical manner to eliminate its 9 and 19 year modulations consists in using 19 year running means. We show the effects of the running means on the data of 1947-1979 in Table 1 and in Fig.2, having extended the calculations to the other major components N_2 and S_2 (1 cycle/12h): the results cover only the central years 1956 to 1970 because of the data consumed by the means. The trends in the amplitudes of M_2 and S_2 emerge sharply; the one for N_2 is not as clear cut. Our data suggests a decreasing trend for it but Fig.2 shows marked oscillations around it. The average N_2 calculated by Doodson for 1894-1917 amounts to 61.9cm while the later data gives 62.0cm, an almost identical value. The annual samples of N_2 are also disturbed by interactions with M_2 and third order effects: so we conclude that we cannot establish clear trends for the N_2 component. The filtered samples of S_2 follow the regression curve closely:

TABLE 1

Secular trends in 19 year running means of the major components of the tide at Saint John, New Brunswick

Central years: 1956 to 1970 (based on the years 1947 to 1979)

	N_2*		M_2		S_2	
	Amplitude	Phase lag	Amplitude	Phase lag	Amplitude	Phase lag
r	-0.79	+0.39	+0.93	+0.61	-0.99	+0.87
trend	-2.4cm/c	+0.62°/c	+14.5cm/c	+0.72°/c	-4.6cm/c	+2.1°/c
A	62.0cm	313°	303.1cm	342°	49.5cm	17°
2,000	61.1cm	313°	307.9cm	342°	47.9cm	18°

r: coefficient of correlation with the year number

trend: rate of change per century

A: average over 1947-1979

2,000: extrapolation to the year 2,000

* After the 9 year modulation was removed by least squares

M_2: average from Doodson (1924) 1894-1917 = 297.2cm 341.9°

average from Godin and Gutierrez(1986) 1947-1979 = 303.1cm 342°

Rate of increase in M_2 amplitude from the two series: +10.4cm/century

---

Secular trend in the annual mean level Z_o 1947-1979: +21.5 cm/century (r=+0.63)

therefore a downward trend in amplitude of 4.6cm/century, is present. The application of similar statistical tests of significance to the M_2 samples sets too wide limits of confidence on the slope, because of the oscillations; since the mean amplitudes of M_2 in the series of 1894-1917 and 1947-1979 differ markedly, the upward trend they reveal has a solid physical foundation, regardless of the statistics. We remedy the situation, as shown in the RHS of Fig. 2 by replacing the 19 year running means by 20 then 21 year running means; the 21 year running mean removes almost all of the oscillations. The M_2 amplitude can then be represented by the quadratic function:

Mean amplitude of M_2 at Saint John $= 302.85 + 0.2243y + 0.0131y^2$ cm (1)

where y=year with 1963 as a time origin (y=0). The results from the fit to the 21 year running means are retained for the first two terms of (1); the coefficient of the third term has been reduced by the factor 0.7674 (whose limit is 0.75 for a large number of samples) because the sum of y^2 divided by 21 gives $1.3030y_o^2$, y_o being the central year around which the mean is calculated [the term in y^2 is a nonlinearity which must be accounted for in formula (1) for calculating the amplitude of M_2 through time]. The fit (1) to the 21year means of M_2 has a standard deviation of 0.0023 cm/year with 10 degrees of freedom. In this case, the rise in amplitude is seen to be accelerating during the last years. Table 1 gives a rate of 10.4cm/century from the means of the two series considered, a rate of 14.5 cm/century from the later data, while the 21year series (RHS of Fig.3) shows an initial rate of 7 cm/century and a final rate of more than 30 cm/century. All these suggest an acceleration in the rate of increase of M_2.

We are now in a position to understand why Doodson found a decreasing M_2 amplitude

while later observations point in the opposite direction. First he applied the conventional nodal f corrections to it, not being aware that this was inappropriate. We confirm this by showing in the following inset the relevant passage in his paper: what he calls "harmonic constants" are H=R/f and K=X+u. R is the estimate amplitude and X, the phase lag for the given year; f and u are the nodal corrections.

The remaining process of analysis is the utilisation of equations (1) yielding

$$H = R/f \qquad \kappa = \chi + u$$

and these are tabulated as the harmonic constants for the constituent.

For our purposes we transform H, κ to R, χ and then calculate R cos $(\chi - \chi_0)$, R sin $(\chi - \chi_0)$.

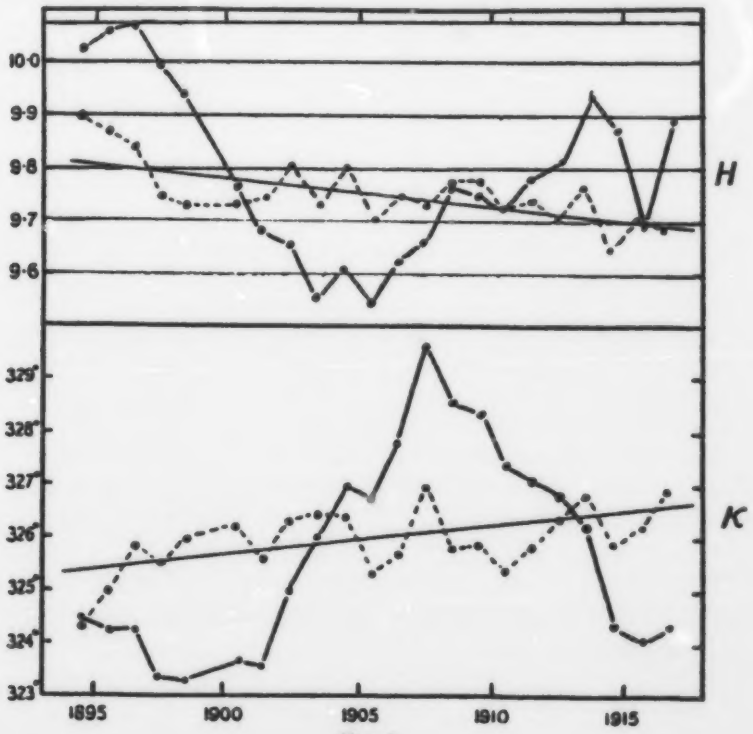


FIG. 1.

Figure 3. Harmonic Constants: M_2, St. John, N.B.
——— original data (amplitudes in feet)
- - - - - - data after reduction.
——— mean secular changes.

His "Fig.1", p.518 of his paper (shown in the preceding inset), shows peak (corrected) M_2 amplitudes around the years 1897 and 1913; astronomy gives the values of the correction factor for the amplitude modulations f=0.9729 and 0.9643 for these years, corresponding with years of minimum M_2 amplitude as revealed by an annual analysis before the correction. Therefore the corrections distorted his results: one must deal exclusively with the raw values of the harmonic components of the tide at Saint John, for the semidiurnal band. We reconstructed the raw

TABLE 2. **M_2 SAMPLES 1894 to 1917**

Amplitudes obtained from scaling "Fig.1" in Doodson(1924) [shown in preceding inset] and multiplying by f. The correction f is obtained from the Foreman programs for tidal analysis.

Year	From diagram	Multiplied by f (raw)			
1894	10.03	9.67	1906	9.54	9.82
1895	10.09	9.73	1907	9.62	9.80
1896	10.10	9.77	1908	9.66	9.73
1897	10.00	9.72	1909	9.77	9.73
1898	9.94	9.76	1910	9.75	9.61
--			1911	9.73	9.48
1901	9.77	9.97	1912	9.78	9.46
1902	9.69	9.98	1913	9.81	9.46
1903	9.66	10.01	1914	9.94	9.59
1904	9.55	9.92	1915	9.88	9.56
1905	9.61	9.96	1916	9.70	9.46
	Units in feet		1917	9.90	9.77

annual values from his figure 1 and the known values of f in Table 2.

Raw values of M_2 are not very useful to search for trends, as seen in our Fig.1, the trends found depending as much on the specific years utilized as on the fluctuations in the data. Therefore we also subjected Doodson's raw data to 19 year running means: these supplied four samples, exhibiting a slight downward trend. This is analogous to the situation encountered in Fig. 2 between the central years 1960 to 1963: had we had only 23 years of observations, we would have concluded, just like Doodson, that the M_2 tide appears to be decreasing at Saint John. **Therefore at least 30 years of observations are needed before discerning well defined patterns.**

To complete our search for trends, we used the least square fit to M_2 shown in Fig.1 to remove the cyclical portion from the records, hoping to reveal from the remainders, samples containing exclusively the trends for all the years of observations. These have a correlation of +0.37 with time and yield a rate of increase in the M_2 amplitude of +8.7 cm/century. Since they are very irregular, this makes it obvious that running averages become mandatory if we wish to extract more precise estimates.

FIT OF THE M_2 SAMPLES AT SAINT JOHN BETWEEN THE YEARS 1947–1979

We treat the annual vector samples (H,g) on the amplitude and Greenwich phase lag (Z=+4) of M_2 in Fig.1, as being represented by a vector $[H_o(y),g_o(y)]$, varying linearly with y, and a pair of modulating vectors $[H_1,g_1(y)]$ and $[H_2,g_2(y)]$, (y=year) using the modulating frequencies d and d_1. Doodson chose d=1 cycle/18.6years (↔19.33°/ years), linked with the progression of the Moon's nodes around the ecliptic, and d_1=2d; he felt he needed to introduce the extra modulation from the periodicities observed. We take the same d as Doodson since it definitely is the correct one but use d_1=1 cycle/8.85years(↔40.68°/years), created by the variation

of the Moon's eccentricity. From a numerical viewpoint, the data available cannot differentiate between 2d and our d_1. A least square fit to the M_2 samples covering the years 1947-1979, with the model in polar form:

$$M_2 = [H_o(y),g_o(y)] + (H_1,dy-a) + (H_2,d_1y-b) + \text{error} \qquad (2)$$

the first term denoting the M_2 fitted by the secular trend given in Table 1, yields:

$$M_2 = [303.3+0.145y,341.84^\circ+0.0072^\circ y] + (7.67,dy-32^\circ) + (1.28,d_1y-245^\circ) \pm (2.3cm,0.9^\circ) \qquad (3)$$

having centered the trends and modulations around the year 1963 ($\leftrightarrow y=0$). The last term is the standard deviation of the difference between the fitted H or g and the observations. We show the fit in Fig.1. It supplies four sample extremes in H whose mean is equivalent to a modulation of $\pm 2.6\%$, when referred to the amplitude extrapolated to the given year, using the results of Table 1. The observations themselves have fluctuations whose average is $\pm 2.8\%$; in the case of a linear response to the applied forces, the range of modulation should be some $\pm 3.7\%$. The standard deviation of the error introduces an uncertainty of at least $\pm 0.7\%$. The term contributed by the 9 year modulation falls below the level of variability. We retained the decimals simply to differentiate between the estimates. The data allows to state at most that:

$$\pm 2\% < \text{range of } M_2 \text{ modulation at Saint John} < \pm 3\%. \qquad (4)$$

The fragmentary observations available for Yarmouth (43°50'N 66°07'W), located near the

mouth of the Bay of Fundy, spread between 1968 and 1978, yield a smoother succession of M_2 samples. These too reveal a reduced range of modulations of some $\pm 2.6\%$, indicating that noticeable frictional effects are

also at work in the Gulf of Maine itself.

IRREGULARITY OF THE SIGNAL

The analyses scrutinized by Doodson had been carried out by Roberts (1870), using hand labour and computational shortcuts. His "Fig.1" (Doodson 1924) [inset on p.10] reveals irregular amplitudes for M_2, the data being very disturbed around 1910 and 1916. The results of our analyses displayed in our Fig.1, derived from a strict least square fit and using a computer, also show an irregular M_2. We investigated further the irregularity of the tide at Saint John by computing, on an hourly basis, the difference between the level observed and the predicted tide, using the harmonic components valid for the specific year and without nodal corrections, in order to minimize the effect of rapidly changing components and of erroneous corrections. Plots of the differences showed that even with these "best fit" predictions, much unpredictability still remains in the signal reaching Saint John, some of it associated with the range of the tide, but not always. Therefore the irregularities present in our Fig.1 are authentic and do not reflect a failure of the analysis process. It does not make sense under the present circumstances to seek precise numbers for anything associated with the local tide: we should satisfy ourselves with orders of magnitude.

The results for St. John, N.B., and for Bombay, Apollo Bandar, are as follows :—

	St. John, N.B.	Bombay, A.B.
H_0	9·752 feet	4·008 feet
H_1	0·336 ,,	0·155 ,,
H_1'	0·147 ,,	0·018 ,,
κ_0	325°·93	330°·6
κ_1	332°·7	327°
κ_1'	355°·4	63°

(12)

INTERPRETING DOODSON'S FORMULA FOR THE EVOLUTION OF M_2

Doodson introduced the concept of a conjugate component i.e. a mirror image of a satellite component. In our case, the satellite of M_2 has frequency σ_0- d, so the conjugate has frequency σ_0+d (σ_0 is the frequency of M_2). He correctly ascribed its presence to quadratic friction which creates triple interactions between the tidal components. One of the possible results of the interaction of σ_0 with another component σ_1 is the new frequency $2\sigma_0$-σ_1; in the case of M_2 and its satellite, the new frequency created becomes σ_0+d, the conjugate. His formula (12) for M_2 was applied not to the raw M_2 annual samples but to the overcorrected values. The two satellites have relative amplitudes of 3.45 and 1.51%; their resultant would give a range of modulations covering $\pm$1.9% to 5.1% at Saint John. They can be only viewed therefore as fitting constants, for the time series available to Doodson, since physics shows that it should always be less then $\pm$3.7%. From the years 1896 to 1903, we calculate from Table 2 a sample range of 2.4% and from 1903 to 1912, another sample of 5.8%, whose average supplies an estimate of $\pm$2.0% for the amplitude modulation. Doodson utilized all the sample years to calculate his estimate, although only the interval 1894 to 1908 seems trustworthy; yet this interval would supply an estimate of $\pm$1.2%.

Therefore, as already stated, it is impossible, even from the more recent data, to estimate the parameters of the tide reaching Saint John with any degree of precision. We show in the following inset a facsimile of a Table published in **SCIENCE** by Ku, Greenberg, Garrett and Dobson (1985) which quotes some numbers for the range of modulation at Saint John based on the data just surveyed: the discussion of error, alluded to in note (7) is of particular interest.

Table 1. Results from tidal analyses and model trials. For a discussion of error limits see (7).

Data	R' (%)			δ' (deg)	a (deg)	b (deg)		
	Analysis	Model 3.73	Model 4.0	analysis	analysis	Analysis	Model 3.73	Model 4.0
Astronomical	3.73			0	2.14	0		
Halifax (1920–1980)	3.64			0.4	2.08	0.01		
Saint John (1947–1971)	2.30	2.15	2.30	−12.0	2.22	−0.18	−0.13	−0.14
Saint John (Doodson) (1894–1916)	2.40			28.6	2.71	−0.19		
Bar Harbor (1947–1966)	2.35	2.39	2.56	3.6	2.39	−0.11	0.02	0.02
Boston (1947–1966)	2.61	2.55	2.74	6.8	2.52	−0.34	0.04	0.04
Transport		2.36	2.53				0.03	0.03

There also exists a marked phase difference between M_2 and its satellite. Linear theory would put a near zero phase difference between the two, while quadratic friction automatically creates a discontinuity in phase between two close components, in the resolved spectrum of a nonlinear process (Godin 1989). The two conjugate components coalesce into a single contributor in our formula (2) since we measure the phases from $\sigma_o t$; a 19 year analysis would reveal them as separate entities however. Such a type of analysis will not be fruitful for Saint John since the local spectrum is continuous, on account of the fluctuations in the signal.

We have evidence that M_2 and N_2 interact strongly (Godin 1988), although this affects N_2 much more than M_2; since N_2 has a 9 year modulation, some of it might be transfered to M_2. Our Fig. 2 does exhibit something akin to 9 year periodicities in the amplitude of the major components of the tide.

DISCUSSION

The existence of trends, third order effects and of irregularities in the tide of Saint John noted by Doodson (1924) in the earliest data available, has been fully confirmed and clarified from later observations. Third order effects were detected at other sites since (Cartwright 1975, Godin and Gutierrez 1985) and should be detectable just about anywhere provided one searches for them. Sharp and persistent irregularities in the tidal signal however, are specific to Saint John. While it is not possible to give precise values for any of the variables involved, trends are still discernible in the components M_2 and S_2. We summarize in Fig.4 the information which is presently available for M_2. We show in it the average M_2 amplitudes for the time series 1894-1917 and 1947-1979 (squared dots). We also plot the 21 year running mean which are available for the later years. The accordion arrows indicate the intervals during which the Saint John tidal records remain unsurveyed and yet inaccessible to modern data processing and scientific studies. Saint John being the only station inside the Bay of Fundy for which we have long series of observations, we have no direct evidence to help decide if this is a local phenomenon or a reflexion of an overall evolution of the tide reaching the upper regions of the Bay of Fundy. We took care to insert below Table 1, the rise in mean level found at Saint John; we definitely know that the level is rising everywhere in the Western Atlantic (Barnett 1983), so the detected rise in level is not local. We therefore hypothesize that we are in presence of a rapid evolution of the tide of the Bay of Fundy,

the changes being even more marked upstream of Saint John.

This hypothesis is supported by the following physical considerations. The basin of the Bay of Fundy is near resonant to tidal inputs of semidiurnal frequencies (Proudman 1953). In the case of resonance, the amplification at the head of the basin is critically dependent on the depth distribution in its shallower portions: a slight change in it will be reflected in the response of the near resonant frequencies, in our case M_2 and N_2. We confirm this by a quantitative example. We consider a basin whose tide cooscillates with that of the ocean at its mouth and having a depth variation of 4,000m to 16.75m over a distance of 600 km; with a rise in sea level of 25cm, the depth in the upper reaches will become 17 m within a century. The amplification of the tidal input at the head of the basin is calculated from the solution to the equations of hydrodynamics:

$$\frac{\delta u}{\delta t} = -g\frac{\delta Z}{\delta x} - ru \quad (5) \qquad \frac{\delta (Hu)}{\delta x} = -\frac{\delta Z}{\delta t} \quad (6)$$

u:current Z:vertical displacement H:depth r:linearized coefficient of friction t:time x:distance. Taking an exponential variation in depth $H=H_o e^{ax}$, (5) and (6) leads to a differential equation for the vertical displacement Z:

$$\frac{d^2Z}{dx^2} + a\frac{dZ}{dx} + \frac{\sigma^2(1-ir/\sigma)Z}{gH(x)} = 0 \quad (7)$$

for oscillatory motion $u,Z \sim e^{i\sigma t}$, which has the general solution:

$$Z(X)=(H_o/H)^{1/4}[AJ_1(X)+BN_1(X)]e^{i\sigma t} \quad (8)$$

$$u(X)=(H_o/H)^{1/4}[g/H_o(1-ir/\sigma)]^{1/4}[AJ_o(X)+BN_o(x)]e^{i(\sigma t-1/4\pi)} \quad (9)$$

having defined $X=(2\sigma/a)[(1-ir/\sigma)/gH(x)]^{1/4}$. J_r and N_r are Bessel and Neumann functions of order r; A and B are arbitrary constants of integration. We obtain the specific solution to our problem

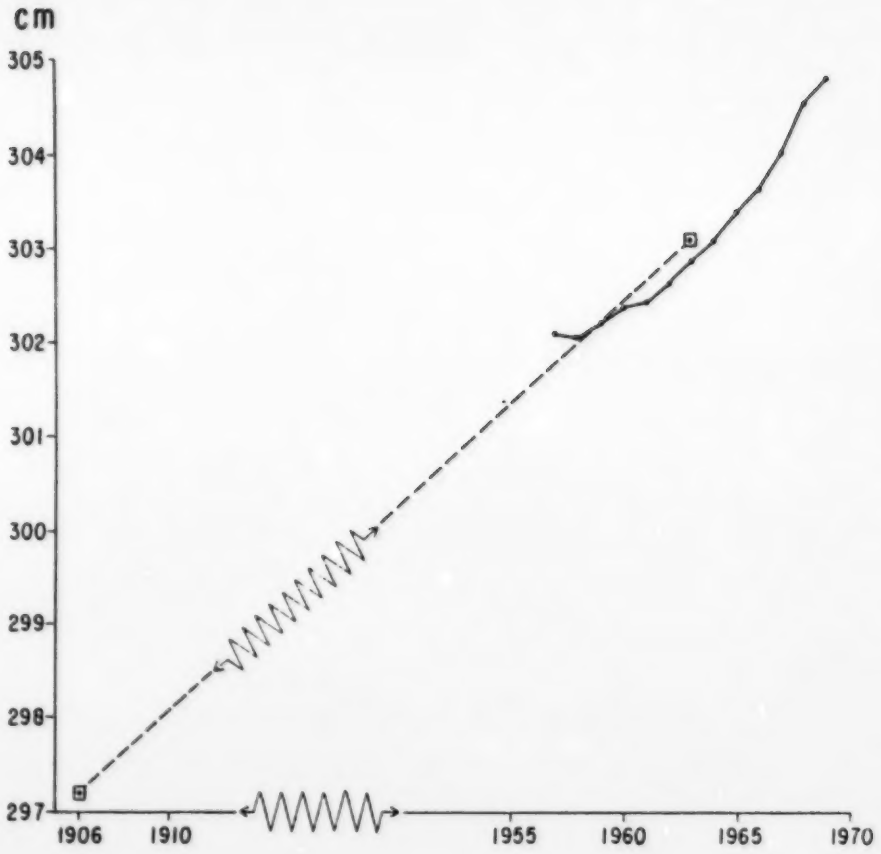


Figure 4. Information available on the secular increase in the amplitude of M_2 at Saint John. Squared points: average amplitude of M_2 for the intervals 1894-1917 and 1947-1979. Dots: 21 year running averages on the data available over 1947-1979. The accordion arrows mark the interval for which the tidal records are yet inaccessible to modern data processing.

by imposing the boundary conditions:

$$Z(X_1) = Z_o \qquad (10)$$

$$u(X_o) = 0 \qquad (11)$$

X_o and X_1 denoting the argument at the head and at the mouth of the basin; Z_o is the amplitude of the oceanic tide at the mouth. The final form of the solution for Z becomes:

$$Z(X) = Z_o (H_1/H)^{1/4} \frac{[J_1(X)N_o(X_o) - N_1(X)J_o(X_o)]}{[J_1(X_1)N_o(X_o) - N_1(X_1)J_o(X_o)]} \qquad (12)$$

TABLE 3.

Values of Bessel functions in a basin whose depth varies from 4,000m to 16.75 and then to 17m over 600 km.

4,000 m to 16.75m: $a=0.912609\times10^{-3}$/km		17m: $a=0.910139\times10^{-3}$/km
$N_o(X_o)$	0.510926,006.9°	0.512074,006.1°
$J_o(X_o)$	0.031178,089.9	0.031790,079.3
$J_1(X_o)$	0.520096,001.4	0.522494,001.4
$J_1(X_1)$	0.077572,358.6	0.077781,358.6
$N_1(X_o)$	0.106110,347.7	0.101552,343.9
$N_1(X_1)$	4.178931,181.4	4.202232,181.4

$$| Z^*(X_o) | / | Z(X_o) | = 32.00/29.75 = 1.076$$

We are interested in the change in amplification $| Z(X_o)^* | / | Z(X_o) |$ at the head of the basin, + denoting the situation after the local level has risen by 25cm. The Bessel and Neumann functions must be calculated from their series expansions for the complex argument: $X=(2\sigma/a)[(1-ir/\sigma)/gH(x)]^{1/4}$. Taking $\sigma=1.405\times10^{-4}$ s^{-1} corresponding to the frequency of M_2,

$r/\sigma = 1/20$, we find:

$$| Z(X_o)^* | / | Z(X_o) | = 1.076$$

an increase of 7.6% in the amplification at the head of the basin for a rise of 25cm in sea level; the details of the calculations are found in Table 3. The depth variation 4,000m to 17m over 600 km, endows the basin with a near quarter wavelength for the M_2 input, the necessary condition for resonance. If the amplitude of the component M_2 was 300cm at the beginning of the century, it will become 323cm at the end of it. Therefore a slight change in sea level suffices to alter the tide in a basin resonant to a given input. The situation at Saint John suggests that the resonant period of the basin is getting closer to that of M_2 and moving away from N_2; S_2 being more distant from it, should be less affected by the change in level, but it should show some increase. Yet Table 1 and Fig.3 indicate that S_2 is falling: we attribute this negative trend to its frictional interaction with M_2. In the case of quadratic friction, the dominant component in the band damps its lesser components more than they damp it; at Saint John, the observations show a clear correlation between the maxima of M_2 and the minima of S_2 (negative correlation) because of this effect [Fig.7 in Godin and Gutierrez 1986]. The trend revealed for S_2, a component having a magnitude of only 50cm compared to some 300cm for M_2, reflects this negative correlation with the increase in M_2 more than its own enhancement.

Associated with the rise in sea level, will be a slight modification of the fields of currents and of the patterns of sedimentation, which will modify further the response characteristics of the basin. Therefore we view the trends noted in the tide of Saint John, as a consequence of the increase in mean level and of its near resonant response to the oceanic input.

Our conclusions are based on the scrutiny of the tidal recordings for Saint John over the interval 1947 to 1979 and on Doodson's 1924 paper. Although our records extended till 1981, we had rejected those for the years 1980 and 1981 because of their poor quality. For the sake of rounding off our study, we obtained from D.G. Mitchell, of the Marine Environment Data Service in Ottawa, the hourly heights for Saint John compiled till 1988. Our inspection revealed numerous failures and gaps, some exceeding one month in width, between the years 1978 and the spring of 1986 as shown in Fig.5. Since continuity in the records is essential for the present investigation, we still carried out harmonic analyses on the fragments available. M_2 exhibited a sharp and rapid downturn in amplitude between 1978 and 1988; its phase lag, adjusted for nodal modulations, showed a steady increase over the same interval, amounting to some 33°/century, while it oscillated around its mean value of 342° over the previous 33 years. S_2, which originally had a neat inverse correlation with M_2, stopped having such a correlation over the last 11 years. We interpret such anomalies in the following way. An instrument is interrupted only after it shows evidence of malfunction: consequently some of the recordings which make their way into the files, are tainted to some degree. Now that the gauge for Saint John is functioning properly, we shall need to accumulate thirty more years of good quality data before being in a position to resume the investigation we initiated.

It would be of interest in the mean time to document the exact position where the tide gauge has been installed since the days of Dawson in 1894. We have evidence (Godin 1991b) that the tide varies most rapidly between the entrance to the Saint John river and the sill which marks the end of the Reversing Falls; slight shifts in the position of the gauge will definitely cause a marked change in the tide being recorded.

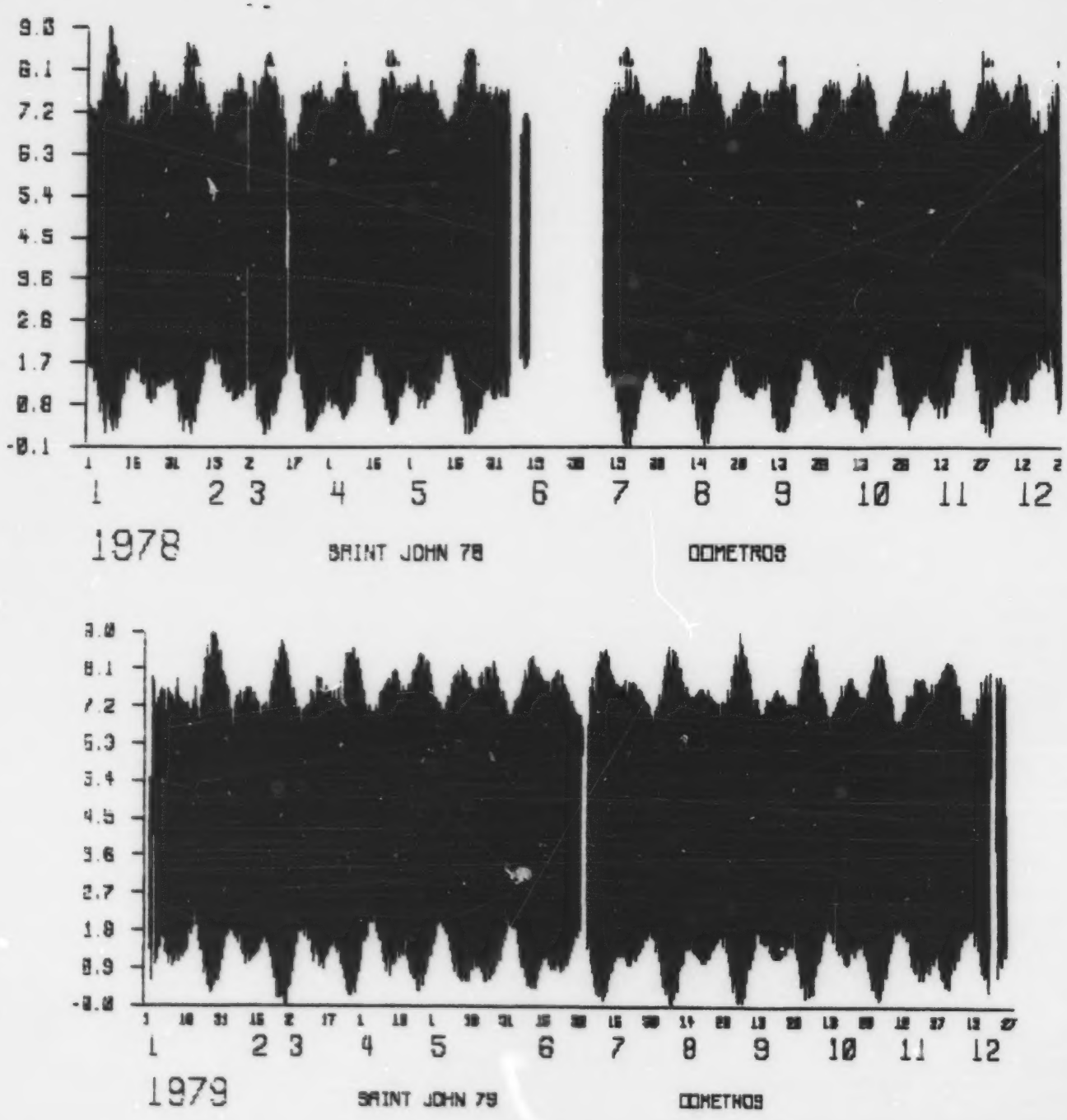
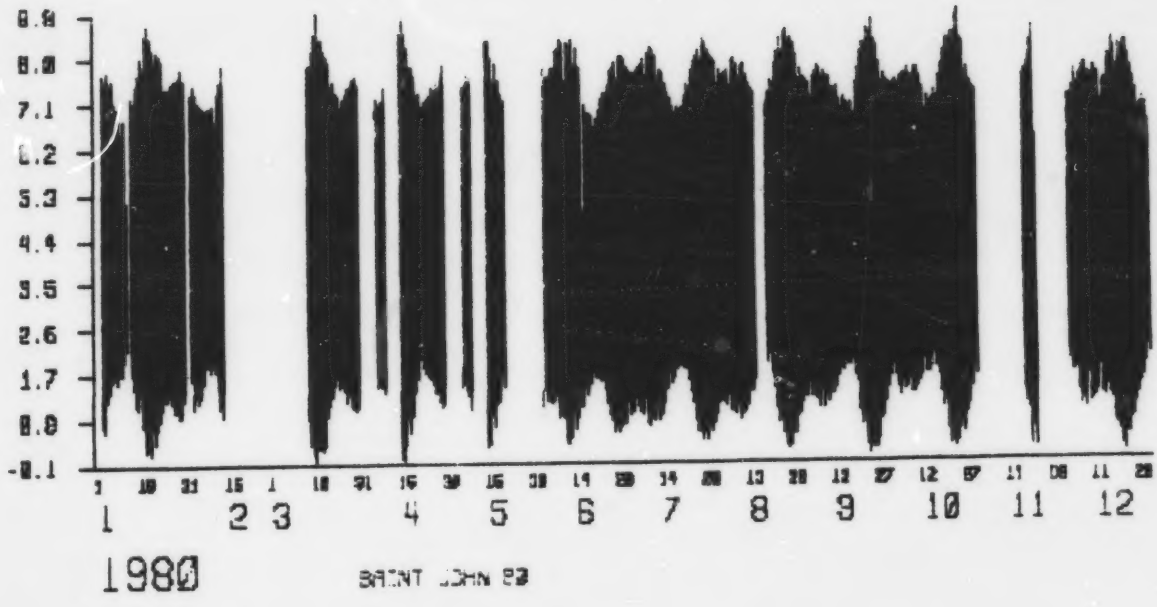
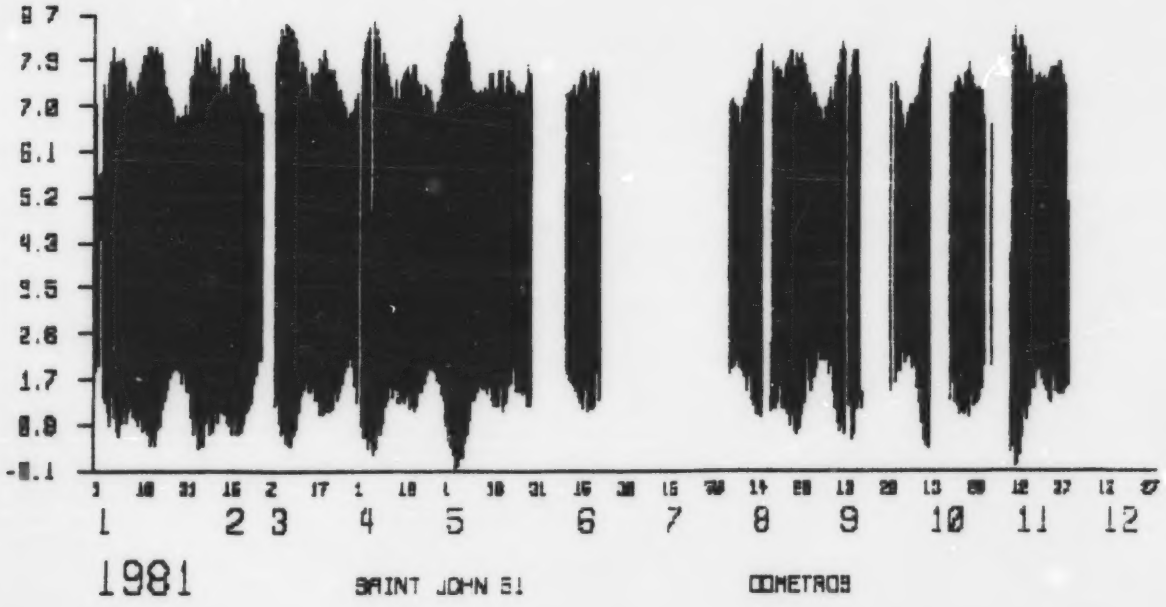
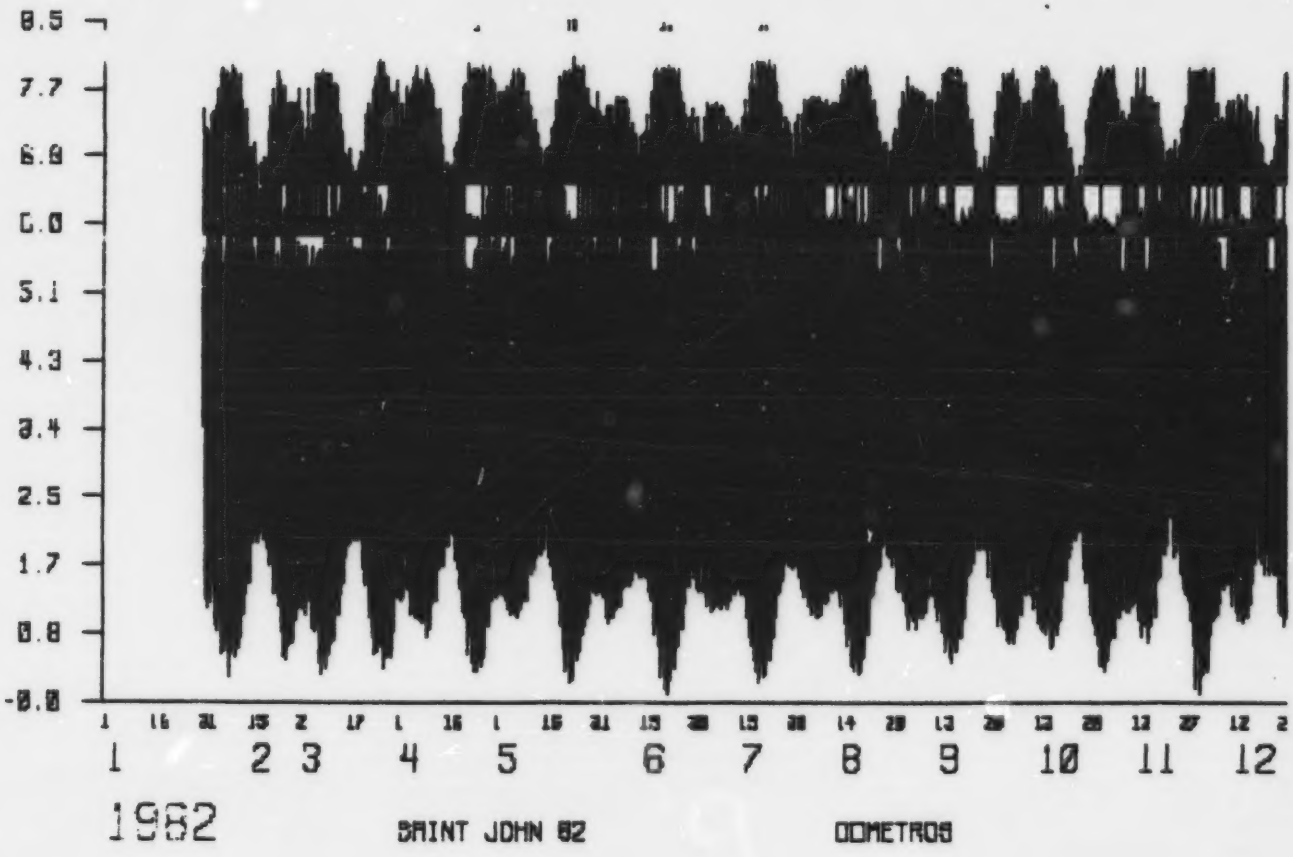


Fig. 5. Compressed view of the tidal records available for Saint John between the years 1978 to 1986.


1 2 3 4 5 6 7 8 9 10 11 12
1980
SAINT JOHN 80


1 2 3 4 5 6 7 8 9 10 11 12
1981
SAINT JOHN 81
CENTIMETROS


8.5
7.7
6.8
6.0
5.1
4.3
3.4
2.5
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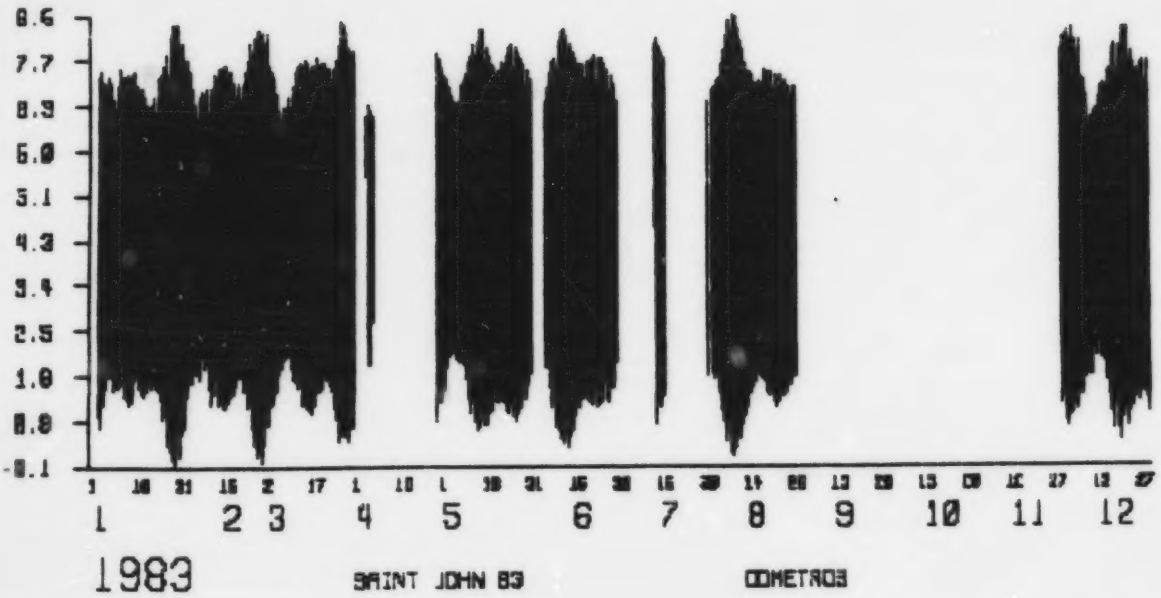
0.8
-0.0
1 2 3 4 5 6 7 8 9 10 11 12
1982
SAINT JOHN 82
ODOMETROS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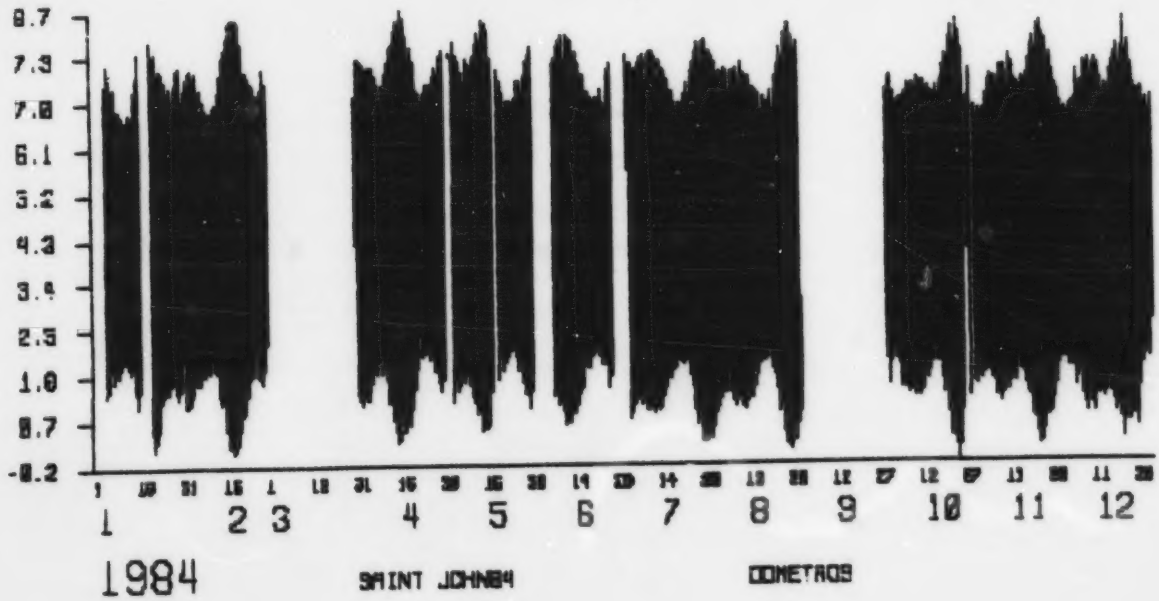
1983
SAINT JOHN 83
COMETROS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1.6
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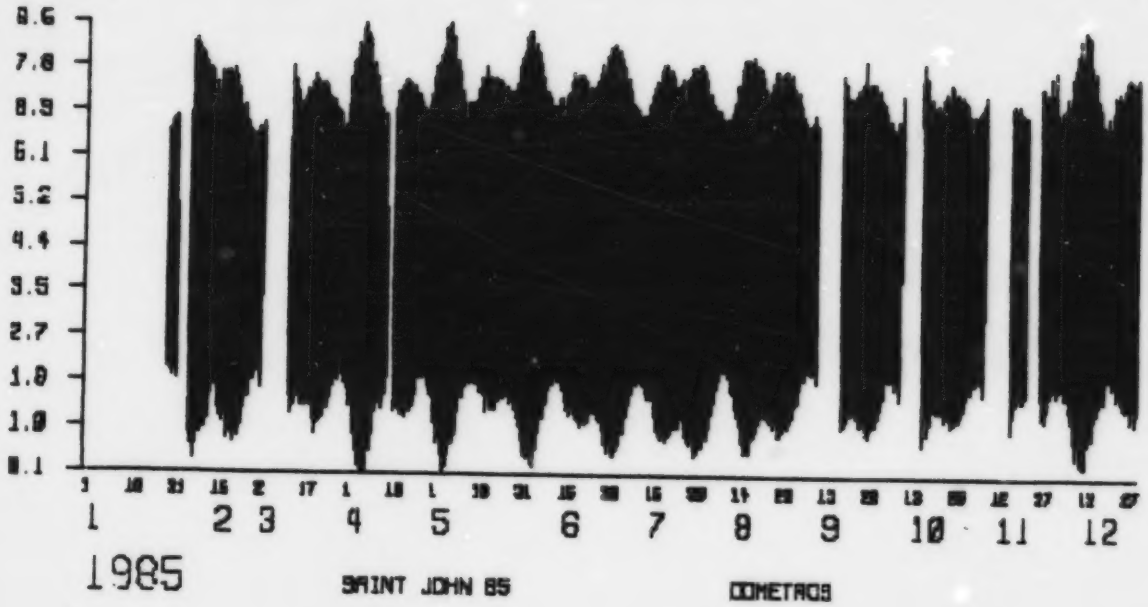
1 2 3 4 5 6 7 8 9 10 11 12
1984
SAINT JOHN84
COMETROS


8.6
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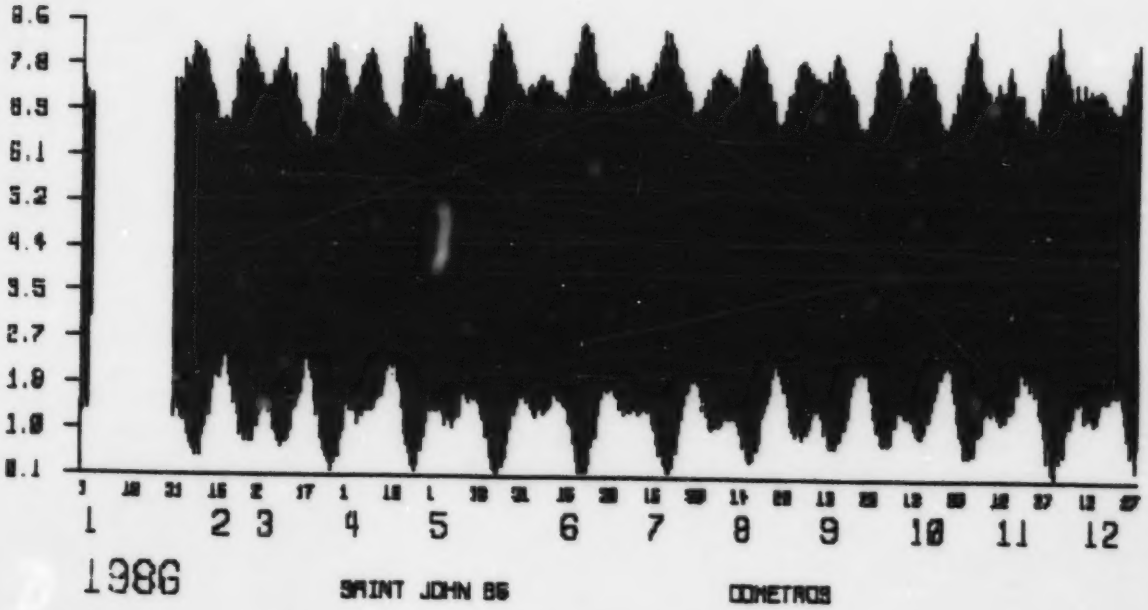
1 2 3 4 5 6 7 8 9 10 11 12
1985
SAINT JOHN 85
COMETAS


8.6
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4.4
3.5
2.7
1.8
1.0
0.1
1 2 3 4 5 6 7 8 9 10 11 12
1986
SAINT JOHN 86
COMETAS

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